Reply to the comment by M A Almeida *et al*

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COMMENT

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Abstract. We reply to the comment on the Lie symmetries of the 2D Lotka–Volterra system.

The comment by Almeida *et al* [1] pertains more to [2] cited in their paper rather than to our paper given in [1], where we have given a short discussion in section 4.1 on the existence of infinite dimensional Lie algebras corresponding to time-dependent symmetries for the two-dimensional Lotka–Volterra (LV) equation. In any case we wish to make the following observations. The existence of a time-independent integral of motion

$$I_2 = x^{-a} y^b \mathrm{e}^{-(x+y)} \tag{1}$$

for arbitrary values of the parameters a and b for the LV equation is well known (see for example [2]) and so naturally the fact that the corresponding time-independent symmetry is transcendental is also known. What is more important is to find other interesting symmetries, which as found in [2] and [1] cited in the 'comment' turns out to be time-dependent and polynomial in the variables x and y, giving rise to the time-dependent integral

$$I_1 = \mathrm{e}^{-at}(x+y) \tag{2}$$

but only for the specific parametric choice $(a+b) = 0$ and not for general choices of a and b.

An immediate consequence of the existence of both the integrals I_1 and I_2 for the Lotka–Volterra equation for the choice $(a+b) = 0$ is that an explicit solution can be given as

$$x = \frac{I_1 \mathrm{e}^{at}}{1 + I_2^{(1/a)} \exp(I_1 \mathrm{e}^{at}/a)}$$

and

$$y = I_1 \mathrm{e}^{at} - \frac{I_1 \mathrm{e}^{at}}{1 + I_2^{(1/a)} \exp(I_1 \mathrm{e}^{at}/a)} \tag{3}$$

for the above parametric choice. For the general case such an explicit solution does not seem to be possible.

References

[1] Almeida M A, Moreira I C and Ritter O M 1996 *J. Phys. A: Math. Gen.* **29** 1141
[2] Minorsky N 1962 *Nonlinear Oscillations* (Princeton, NJ: Van Nostrand)

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